Chapter 4

Multi-Echelon Data Envelopment Analysis Variable Returns to Scale Models for Performance Evaluation of Supply Chains

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/intechopen.78072

Abstract

This paper develops a variable returns to scale multi-echelon data envelopment analysis (DEA) model to measure the efficiency of supply chain. The model is constructed at first with the assumption of serial sequence in a supply chain. The inputs of one stage become the output of the other stage in the multi-echelon structure. The traditional variable returns to scale model of DEA is modified to fit in the multi-echelon structure. The developed model helps to evaluate the supply network in a coordinated manner. It also provides helpful insights as how to improve the supply network performance.

Keywords: DEA, variable returns to scale, supply chain, process cycles

1. Introduction

Supply chain is a coordinated system of various processes meshed together to form a network of strategic decision making. All the stages in supply chain are connected together through feed-forward flow of materials and services as well as feedback flow of information [1]. Several studies in the literature have underlined the practical importance of supply-chain performance measures. Gunasekaran and Patel [2] argue that frequent evaluation and benchmarking of supply chain outputs are necessary for companies to achieve their supply chain management (SCM) objectives. Supply chain measures are crucial for the coordination of cross-functional and inter-organizational activities in SCM, and for forming long-term alliances among firms in the chain [3, 4]. Performance evaluation of supply chain helps to improve processes and coordinate efforts of different stages and make contracting and risk sharing feasible in a supply chain. However, detailed analysis of processes in a supply chain is a time and resource consuming process [5]. To improve supply chain performance it is imperative to measure it. With the measurement of performance the symptoms of the problem in a supply chain can be identified. After identifying the symptoms managers can focus on detailed activity analysis at an operational level.

Bibliography of [6] reveals that there is dearth of literature that utilized mathematical programming and associated statistical techniques to help decision making in supply chain benchmarking. Reference [7] reveals most models whether deterministic or stochastic deals with single player in a supply chain rather than considering supply chain as a system. There are some issues in measuring the efficiency of supply chain. The first point is the involvement of different stages of supply chain to contain the DEA model. Secondly, the improvement projection has to be coordinated at all stages of supply network.

New methods have been developed to measure the supply chain performance using DEA. For instance, Ref. [8] decomposes the traditional DEA model to product of efficiencies by decomposing the overall efficiency score. Reference [9] also decomposes the overall efficiency scores of multiplicative efficiency model using game theory concepts. Reference [10] presents a model to decompose overall radial efficiency of supply network at additive weighted average of all the individual stages of supply chain network. In many cases, DMUs may have internal or network structures; see for example, [11–13]. The types of special DMUs have inputs converted to outputs and vice versa in the intermediate stages. Recently, some of the studies have modeled DEA in two-stage processes. For example, Ref. [14] divides the US commercial banks into profitability and marketability as first stage and second stage respectively. For the first stage, they use labour and assets as inputs and profits and revenues and outputs. In the second stage, the output in the first stage, i.e., profit and revenue are used as inputs and market value, returns and earning per share constitute output. Reference [8] uses the same method of two stage process for non-insurance companies where they use operating and insurance expenses as outputs in the first stage and then underwriting and investment profits in the second stage. Other examples include the impact of information technology use on bank branch performance [15], two stage Major League Baseball performance [16], and many others.

In this paper, we use multi-echelon variable returns to scale Data Envelopment Analysis (DEA) to measure performance of supply chain. In traditional DEA, the internal structures are generally ignored, the efficiency score is a function of given inputs and outputs [17]. More specifically, the production capability of production units is formulated only under some general assumptions.[1] The advantage of DEA is that utilizing multiple inputs and outputs it gives a single index for measurement.

Although there are certain advantages of DEA, however, when dealing with supply chain it becomes a limitation. Therefore, the DEA model needs to be modified appropriately to contain the different connecting stages of supply chain to act as a single DMU. Further, DEA has an assumption that the stages of supply chain are independent and not connected which clearly violated the coordination nature of supply chain. In this paper, we have modeled

[1]These assumptions on the production function include: monotonicity, convexity, envelopment and minimum extrapolation; see [18] for an explanation.

the multi-echelon variable returns to scale DEA in such a way that the coordination property of supply chain is retained.

There is a substantial body of DEA literature, however, the use of DEA in supply chain network to evaluate performance is limited. Reference [19] proposes a DEA model using value chain approach to measure the performance of supply chain stages; Ref. [15] use the value-chain model to evaluate IT's impact on firm performance. References [7] and [20] propose efficiency evaluation approaches for a two-tier supply chain model from a game theoretic perspective. [12, 21, 22] introduce the network DEA model, in which the interior structure of production units can be explicitly modeled. These studies tend to view supply chains as a sequence of static, but independent processes. Reference [8] described a two-stage process where 24 non-life insurance companies used operating and insurance expenses to generate premiums in the first stage and then underwriting and investment profits in the second stage. Other articles in this general area are due to [9, 10, 16]. Reference [23] suggests that performance measures should be systematically deployed in a top-down fashion to ensure the organization is controllable and well coordinated. A significant body of work has been directed at problem settings where the DMU is characterized by a multi-stage process; supply chains and many manufacturing processes take this form [24–26]. Supply chains similarly need a systematic structure of performance measures for different units, e.g., individual firms, tiers in the supply chain, and the whole chain.

DEA models are classified with respect to the type of envelopment surface, the efficiency measurement and the orientation (input or output). There are two basic types of envelopment surfaces in DEA known as constant returns-to-scale (CRS) and variable returns-to-scale (VRS) surfaces. Each model makes implicit assumptions concerning returns-to-scale associated with each type of surface. Charnes et al. [17] introduced the CCR or CRS model that assumes that the increase of outputs is proportional to the increase of inputs at any scale of operation [27]. Banker et al. [28] introduced the BCC or VRS model allowing the production technology to exhibit increasing returns-to-scale (IRS) and decreasing returns-to-scale (DRS) as well as CRS. All the mentioned papers of recent literature has examined a particular form of network structure, namely, where the DMU is a two-stage serial process in which the outputs from the first stage are intermediate variables that serve as inputs to the second stage. The current article extends this idea to include those situations where the overall process can be decomposed into product of the efficiencies of four processes. Therefore, we propose two models of efficiency decomposition that deals with the assumption of variable returns to scale (VRS).

The rest of the paper is organized as follows. In Section 2, we propose two models of efficiency decomposition namely, multi-echelon VRS model and multi-echelon VRS additive model. The proposed models assume variable returns to scale (VRS). In Section 3, we discuss model application. Finally our conclusions are presented in Section 4.

2. Multi-echelon DEA models

It is important to note that traditional DEA models assume that the operations follow constant returns to scale. This represented one of the most limiting factors for the applicability of DEA, at least in the early years. Many economists viewed this assumption as over-restrictive and preferred alternative statistical procedures in spite of the advantage offered by DEA.

Modifications of DEA to handle VRS categories were first described in 1984, when [28] came up with a simple yet remarkable modification to the CCR DEA models in order to handle variable returns to scale. This modification was suggested by comparing some previous studies on production functions. Hence, the DEA model is termed BCC (Banker, Charnes, Cooper) model. In general, DEA programs incorporating an additional convexity constraint to take into account variable returns to scale are called variable returns to scale or VRS model.

2.1. Multi-echelon variable returns to scale model

Consider the c-cycle process pictured in **Figure 1**. Suppose we have n DMUs and that each $DMU_j(j = 1, 2, \ldots, n)$ has m inputs to first stage, and S outputs from this stage, z_{sj_o}, $s = 1, 2, .., S$. These S outputs then become the inputs to the second cycle and Z_{op_o}, where $o = 0, 1, \ldots, O$, is the input or enters as a input of the existing stage and other subsequent stages. The outputs from second, third and fourth stages are denoted as yr_{j_o} where $r = 1, 2, \ldots, R$, v_{lj_o}, where $l = 1, 2, \ldots, L$ and w_{kg_o} where $g = 1, 2, \ldots, G$. The weights of cycle 1, cycle 2, cycle 3, and cycle 4 are η_S^A, u_r, μ_l, and γ_k. The input weights of stage 1, 2, 3, and 4 are v_i, v_{op}, w_{op}, and O_{op}. The VRS efficiency score for the four stages can be determined by the following VRS models [28]:

$$\theta^* = Max \frac{\sum_{s=1}^{S} \eta_s^A z_{sj_o}}{\sum_{i=1}^{m} v_i x_{ij_o}} \cdot \frac{\sum_{r=1}^{R} u_r y_{rj_o}}{\left(\sum_{s=1}^{S} \eta_s^A z_{sj_o} + \sum_{p=1}^{P} v_{op_o} z_{op_o}\right)} \cdot \frac{\sum_{l=1}^{L} \mu_l v_{lj_o}}{\sum_{r=1}^{R} u_r y_{rj_o} + \sum_{q=1}^{Q} w_{op} z_{op_o}} \cdot \frac{\sum_{g=1}^{G} \gamma_k w_{kj_o}}{\left(\sum_{l=1}^{L} \mu_l v_{lj_o} + \sum_{n=1}^{N} o_{op} z_{op_o}\right)} \tag{1}$$

subject to,

$$\frac{\sum_{s=1}^{S} \eta_s^A z_{sj_o}}{\sum_{m=1}^{m} v_i x_{ij_o}} \leq 1 \tag{2}$$

$$\frac{\sum_{r=1}^{R} u_r y_{rj_o}}{\sum_{s=1}^{S} \eta_S^A z_{sj_o} + \sum_{p=1}^{P} v_{op} z_{op_o}} \leq 1 \tag{3}$$

$$\frac{\sum_{l=1}^{L} \mu_l v_{lj_o}}{\sum_{r=1}^{R} u_r y_{rj_o} + \sum_{q=1}^{Q} w_{op} z_{op_o}} \leq 1 \tag{4}$$

$$\frac{\sum_{g=1}^{G} \gamma_k w_{kj_o}}{\sum_{l=1}^{L} \mu_l v_{lj_o} + \sum_{n=1}^{N} o_{op} z_{op_o}} \leq 1 \tag{5}$$

$$\sum_{s=1}^{S} \eta_s^A + \sum_{r=1}^{R} u_r + \sum_{l=1}^{L} \mu_l + \sum_{g=1}^{G} \gamma_k = 1 \tag{6}$$

$$\eta_s^A, v_i, u_r, v_{op_o}, \mu_l, w_{op}, \gamma_k, o_{op} \geq 0$$

http://dx.doi.org/10.5772/intechopen.78072

Using Charnes-Cooper transformation [29], Eqs. (1)–(5) are equivalent to,

$$Max\left[\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}\right)\cdot\left(\sum_{r=1}^{R}u_r y_{rj_o}\right)\cdot\left(\sum_{l=1}^{L}\mu_l v_{lj_o}\right)\cdot\sum_{g=1}^{G}\gamma_k w_{kj_o}\right]$$

subject to,

$$\left(\sum_{i=1}^{m}v_i x_{ij_o}\right)\cdot\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}+\sum_{p=1}^{P}v_{op_o}z_{op_o}\right)\cdot\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}\cdot \left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}\right)=1 \quad (7)$$

$$\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}\right)-\left(\sum_{i=1}^{m}v_i x_{ij_o}\right)\le 1 \quad (8)$$

$$\left(\sum_{r=1}^{R}u_r y_{rj_o}\right)-\left(\sum_{s=1}^{S}\eta_S^A z_{sj_o}+\sum_{p=1}^{P}v_{op}z_{op_o}\right)\le 1 \quad (9)$$

$$\left(\sum_{l=1}^{L}\mu_l v_{lj_o}\right)-\left(\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}\right)\le 1 \quad (10)$$

$$\left(\sum_{g=1}^{G}\gamma_k w_{kj_o}\right)-\left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}\right)\le 1 \quad (11)$$

$$\sum_{s=1}^{S}\eta_s^A+\sum_{r=1}^{R}u_r+\sum_{l=1}^{L}\mu_l+\sum_{g=1}^{G}\gamma_k=1 \quad (12)$$

$$\eta_s^A, v_i, u_r, v_{op_o}, \mu_l, w_{op}, \gamma_k, o_{op}\ge 0$$

A scale efficiency score of less than one does not indicate whether the organization is bigger or smaller than its optimal size. To establish this, an additional variant of DEA, one subject to non-increasing returns to scale must be run. The DEA linear programming problem for the non-increasing returns to scale case is given by:

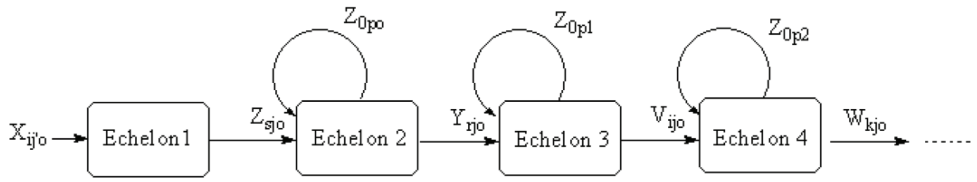

Figure 1. A serial multi-echelon DEA with inputs, *carryover inputs*, and outputs.

$$Max\left[\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}\right)\cdot\left(\sum_{r=1}^{R}u_r y_{rj_o}\right)\cdot\left(\sum_{l=1}^{L}\mu_l v_{lj_o}\right)\cdot\sum_{g=1}^{G}\gamma_k w_{kj_o}\right]$$

subject to,

$$\left(\sum_{i=1}^{m}v_i x_{ij_o}\right)\cdot\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}+\sum_{p=1}^{P}v_{op_o}z_{op_o}\right)\cdot\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}\cdot \left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}\right)=1 \tag{13}$$

$$\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}\right)-\left(\sum_{i=1}^{m}v_i x_{ij_o}\right)\le 1 \tag{14}$$

$$\left(\sum_{r=1}^{R}u_r y_{rj_o}\right)-\left(\sum_{s=1}^{S}\eta_S^A z_{sj_o}+\sum_{p=1}^{P}v_{op}z_{op_o}\right)\le 1 \tag{15}$$

$$\left(\sum_{l=1}^{L}\mu_l v_{lj_o}\right)-\left(\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}\right)\le 1 \tag{16}$$

$$\left(\sum_{g=1}^{G}\gamma_k w_{kj_o}\right)-\left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}\right)\le 1 \tag{17}$$

$$\sum_{s=1}^{S}\eta_s^A+\sum_{r=1}^{R}u_r+\sum_{l=1}^{L}\mu_l+\sum_{g=1}^{G}\gamma_k\le 1 \tag{18}$$

$$\eta_s^A, v_i, u_r, v_{op_o}, \mu_l, w_{op}, \gamma_k, o_{op}\ge 0$$

2.1.1. Multi-echelon VRS additive model

We let $\sum_{i=1}^{m}v_i x_{ij_o}+\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}+\sum_{p=1}^{P}v_{op_o}z_{op_o}\right)+\left(\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}\right)+\left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}\right)=|R|$ represent the total amount of resources consumed by the four-cycle process. The model 1–5 with incorporation of variables ψ^A, ψ^B, ψ^C, and ψ^D becomes

$$Max\left[\frac{\sum_{s=1}^{S}\eta_s^A z_{sj_o+\psi^A}+\sum_{r=1}^{R}u_r y_{rj_o+\psi^B}+\sum_{l=1}^{L}\mu_l v_{lj_o+\psi^C}+\sum_{g=1}^{G}\gamma_k w_{kj_o}+\psi^D}{|R|}\right] \tag{19}$$

subject to,

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$$\frac{\sum_{s=1}^{S}\eta_s^A z_{sj_o}+\psi^A}{\sum_{i=1}^{m}v_i x_{ij_o}}\leq 1 \tag{20}$$

$$\frac{\sum_{r=1}^{R}u_r y_{rj_o}+\psi^B}{\sum_{s=1}^{S}\eta_S^A z_{sj_o}+\sum_{s=1}^{S}v_{op}z_{op_o}}\leq 1 \tag{21}$$

$$\frac{\sum_{l=1}^{L}\mu_l v_{lj_o}+\psi^C}{\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}}\leq 1 \tag{22}$$

$$\frac{\sum_{g=1}^{G}\gamma_k w_{kj_o}+\psi^D}{\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}}\leq 1 \tag{23}$$

$$\psi^A,\psi^B,\psi^C,\ and\ \psi^D,\ free\ in\ sign.$$

$$\eta_s^A, v_i, u_r, v_{op_o}, \mu_l, w_{op}, \gamma_k, o_{op}\geq 0,\quad j_o,\ \ p_o=1,2,\dots,n.$$

Model (19)–(23) is equivalent to

$$Max\left[\sum_{s=1}^{S}\eta_s^A z_{sj_o}+\psi 1+\sum_{r=1}^{R}u_r y_{rj_o}+\psi 2+\sum_{l=1}^{L}\mu_l v_{lj_o}+\psi 3+\sum_{g=1}^{G}\gamma_k w_{kj_o}+\psi 4\right] \tag{24}$$

subject to,

$$\left(\sum_{s=1}^{S}\eta_s^A z_{sj_o}\right)-\left(\sum_{i=1}^{m}v_i x_{ij_o}+\psi^1\right)\leq 0 \tag{25}$$

$$\left(\sum_{r=1}^{R}u_r y_{rj_o}\right)-\left(\sum_{s=1}^{S}\eta_S^A z_{sj_o}+\sum_{p=1}^{P}v_{op}z_{op_o}+\psi^2\right)\leq 0 \tag{26}$$

$$\left(\sum_{l=1}^{L}\mu_l v_{lj_o}\right)-\left(\sum_{r=1}^{R}u_r y_{rj_o}+\sum_{q=1}^{Q}w_{op}z_{op_o}+\psi^3\right)\leq 0 \tag{27}$$

$$\left(\sum_{g=1}^{G}\gamma_k w_{kj_o}\right)-\left(\sum_{l=1}^{L}\mu_l v_{lj_o}+\sum_{n=1}^{N}o_{op}z_{op_o}+\psi^4\right)\leq 0 \tag{28}$$

$$|R|=1 \tag{29}$$

$$\psi^1,\psi^2,\psi^3,\ and\ \psi^4,\ free\ in\ sign.$$

$$\eta_s^A, v_i, u_r, v_{op_o}, \mu_l, w_{op}, \gamma_k, o_{op}\geq 0$$

3. Application

A supply chain consists of all parties involved directly or indirectly in fulfilling a customer request. The supply chain includes not only the manufacturers and suppliers, but also transporters, warehouses, retailers and even customers themselves. A supply chain is a series of processes and can be described as cycle view. A cycle view of supply chain divides processes into cycles each performed at the interface between two successive stages of a supply chain [30].

The cycle view of process is important as it delineates the responsibilities of each player of each stage. The process cycle helps making operational decision as it clearly mentions the roles and responsibilities of each member of the supply chain. To evaluate the performance of supply chain we consider the four cycles namely—customer cycle, replenishment cycle, manufacturing cycle and procurement cycle. The first cycle, i.e., the customer cycle starts at the retailer's site. The customer fills in the demand and the demand is received by the retailer. The cycle initiates as soon as the retailer receives the order from the customer.

From the customer cycle we take two inputs - Technological functionality and Sales order by FTE. The first input is the functionality of the technology in place. This is measured in units of functionality where a higher number indicates more functionality. The second input is sales order by full time employee (FTE). This indicator measures the number of customer orders that are processed by full time employees per day. The outputs for customer cycle are Order fulfillment cycle time and Cycle inventory. Order fulfillment cycle time is a continuous measurement defined as the amount of time from customer authorization of a sales order to the customer receipt of product. On the other hand, Cycle inventory represents the average order quantity amount on hand. The inputs and outputs extracted from customer cycle is displayed in **Table 1**.

The *replenishment cycle* [30] starts at the juncture of retailer or distributor interface and includes replenishing retailer inventory. The replenishment policy consist of decisions regarding when to reorder and how much to reorder. The decisions determine the cycle and safety inventory. The inputs of replenishment cycle are—technological functionality and sales order by FTE. The first input is the functionality of the technology in place. This is measured in units of

Customer order cycle	Description
Inputs	
Technological functionality	The functionality of the technology in place. This is measured in units of functionality where a higher number indicates more functionality
Sales order by FTE	This indicator measures the number of customer orders that are processed by full time employees per day.
Outputs	
Order fulfillment cycle time	It is a continuous measurement defined as the amount of time from customer authorization of a sales order to the customer receipt of product
Cycle inventory	It represents the average order quantity amount on hand

Table 1. Inputs and outputs of customer cycle.

functionality where a higher number indicates more functionality. The second input is sales order by full time employee (FTE). This indicator measures the number of customer orders that are processed by full time employees per day. The outputs of replenishment cycle are—Fill rate, Inventory cycle time and Cycle inventory. Fill rate is the number of items ordered compared with items shipped. Fill rate can be calculated on a line item, SKU, case or value basis. Inventory cycle time is a measure of the Manufacturing Cycle Time plus the time included to deploy the product to the appropriate distribution center and Cycle inventory represents the average order quantity amount on hand. The inputs and outputs of replenishment cycle are given in **Table 2**.

The *manufacturing cycle* [30] occurs at the distributor/manufacturer (or retailer/manufacturer) interface and includes all processes involved in replenishing retailer inventory. The manufacturing cycle is triggered by customer orders/replenishment orders/forecast of customer demand and current product availability in the manufacturer's finished goods warehouse.

The inputs of manufacturing cycle are—Bill-of-materials (BOM), Usage quantity and Independent demand ratio. Bill-of-materials (BOM) is a record of all the components of an item, the parent-component relationships, and the usage quantities derived from engineering and process design. Usage quantity is the number of units of a component needed to make one unit of its immediate parent. Independent demand ratio is for manufacturers that also supply replacement parts and consumables this metric helps to define the percentage mix of demand for an item from independent (outside sources) vs. dependent (inside sources). The ratio is calculated by dividing the unit usage for customer orders by the total unit usage of the item from all sources (work orders, sales samples, destructive testing, inventory adjustments, etc.). The outputs of manufacturing cycle are—Finished product cycle time and End item. Finished product cycle time is the average time associated with finalizing activities, such as: package, stock, etc. and the other output End item is the final product sold to a customer. The inputs and outputs of manufacturing cycle are displayed in **Table 3**.

Replenishment process cycle	Description
Inputs	
Technological functionality	The functionality of the technology in place. This is measured in units of functionality where a higher number indicates more functionality
Sales order by FTE	This indicator measures the number of customer orders that are processed by full time employees per day.
Outputs	
Fill rate	The number of items ordered compared with items shipped. Fill rate can be calculated on a line item, SKU, case or value basis.
Inventory cycle time	Measure of the Manufacturing Cycle Time plus the time included to deploy the product to the appropriate distribution center
Cycle inventory	It represents the average order quantity amount on hand.

Table 2. Inputs and outputs of replenishment cycle.

Manufacturing cycle	Description
Inputs	
Bill-of-materials (BOM)	A record of all the components of an item, the parent-component relationships, and the usage quantities derived from engineering and process design.
Usage quantity	The number of units of a component needed to make one unit of its immediate parent.
Independent demand ratio	For manufacturers that also supply replacement parts and consumables this metric helps to define the percentage mix of demand for an item from independent (outside sources) vs. dependent (inside sources). The ratio is calculated by dividing the unit usage for customer orders by the total unit usage of the item from all sources (work orders, sales samples, destructive testing, inventory adjustments, etc.).
Outputs	
Finished product cycle time	Average time associated with finalizing activities, such as: package, stock, etc.
End item	The final product sold to a customer.

Table 3. Inputs and outputs of manufacturing cycle.

The *procurement cycle* [30] takes place at the interface of manufacturer/supplier and includes the necessary processes to make sure that the materials are available for manufacturing to take place as per schedule. In the procurement cycle, the components are ordered by manufacturer from the suppliers that replenish the component inventory. In this cycle components are ordered precisely once the production set up is finalized by the manufacturer.

The inputs from the procurement cycle are - Purchased item and Direct material cost. Purchased item is an item that has one or more parents, but no components because it comes from a supplier. Direct material cost is the sum of costs associated with acquisition of support material. The outputs of procurement cycle are - On time ship rate and Delivery schedule adherence. On time ship rate is the percent of orders where shipped on or before the requested ship date. On time ship rate can be calculated on a line item, SKU, case or value basis. Delivery schedule adherence is a business metric used to calculate the timeliness of deliveries from suppliers. Delivery schedule adherence is calculated by dividing the number of on time deliveries in a period by the total number of deliveries made. The result is then multiplied by 100 and expressed as a percentage. The inputs and outputs of procurement cycle are displayed in **Table 4**.

3.1. Multi-echelon VRS DEA model results

The efficiency results of the CCR and BCC model are shown in **Table 5** for 11 supply chain sub-processes of a particular product (e.g., detergent). First, the efficient supply chains, in each process cycle are: customer order cycle (1, 4, 7, 9, and 11) replenishment process cycle (1, 2, 5, 6, 8, 11), manufacturing process cycle (2, 4, 6) and procurement process cycle (5, 6, 9). The same table shows the efficiency results of RTS. The RTS efficiency score is calculated as the ratio of CCR efficiency score to BCC efficiency score. **Table 5** indicates that, customer order cycle, the BCC efficient but not scale-efficient process, cycles were operating on an increasing returns to scale (IRS) frontier because they can achieve greater economies of scale if they increase the

http://dx.doi.org/10.5772/intechopen.78072

Procurement process cycle	Description
Inputs	
Purchased item	An item that has one or more parents, but no components because it comes from a supplier.
Direct material cost	Sum of costs associated with acquisition of support material.
Outputs	
On time ship rate	Percent of orders where shipped on or before the requested ship date. On time ship rate can be calculated on a line item, SKU, case or value basis.
Delivery schedule adherence	Delivery Schedule adherence (DSA) is a business metric used to calculate the timeliness of deliveries from suppliers. Delivery schedule adherence is calculated by dividing the number of on time deliveries in a period by the total number of deliveries made. The result is then multiplied by 100 and expressed as a percentage.

Table 4. Inputs and outputs of procurement process cycle.

DMU	Customer cycle			Replenishment cycle			Manufacturing cycle			Procurement cycle			Efficiency	
	CCR[1]	BCC[2]	RTS[3]	CCR	BCC	RTS	CCR	BCC	RTS	CCR	BCC	RTS	CCR	BCC
1	1.00	1.00	CRS	1.00	1.00	CRS	0.08	0.19	DRS	0.63	0.98	DRS	0.677	0.792
2	1.00	1.00	CRS	1.00	1.00	CRS	1.00	1.00	CRS	1.00	1.00	CRS	1.000	1.000
3	0.45	0.49	DRS	0.76	0.82	IRS	0.06	0.09	DRS	0.17	0.18	IRS	0.360	0.395
4	1.00	1.00	CRS	0.45	0.61	IRS	0.46	1.00	DRS	0.64	0.92	DRS	0.637	0.882
5	0.51	0.54	IRS	1.00	1.00	CRS	0.13	0.39	DRS	0.44	1.00	DRS	0.520	0.732
6	0.43	1.00	IRS	0.66	1.00	IRS	0.27	1.00	IRS	0.64	1.00	DRS	0.500	1.000
7	0.97	1.00	DRS	0.69	0.70	DRS	0.02	0.03	DRS	0.12	0.12	IRS	0.450	0.462
8	0.52	0.53	IRS	1.00	1.00	CRS	0.10	0.10	CRS	0.28	0.30	IRS	0.475	0.482
9	0.90	1.00	IRS	0.45	0.95	IRS	0.43	0.75	DRS	1.00	1.00	CRS	0.695	0.925
10	0.74	0.94	DRS	1.00	1.00	CRS	0.01	0.02	IRS	0.09	0.11	IRS	0.460	0.517
11	0.76	1.00	DRS	0.96	1.00	DRS	0.01	0.02	IRS	0.06	0.07	IRS	0.447	0.522

[1]Charnes-Cooper-Rhodes Model.
[2]Banker-Charnes-Cooper Model.
[3]Returns-to-Scale.

Table 5. Multi-echelon VRS model results of supply chains.

volume. For customer order cycle, five BCC-efficient retail chains were operating on IRS and four on decreasing returns to scale (DRS) frontiers. Of the BCC-inefficient supply chains, 64% and 20% were in the IRS region in cycle 1 and cycle 2, respectively. As economists have long recognized, an IRS frontier firm would generally be in a more favorable position for expansion, compared to a firm operating in a DRS region. Note that the concept of RTS may be ambiguous unless a process cycle is on the BCC-efficient frontier, since we classified RTS for inefficient process cycles by their input oriented BCC projections. Thus, a different RTS classification may be obtained for a different orientation, since the input-oriented and the output-oriented BCC models can yield different projection points on the VRS frontier. Thus, it is necessary to explore the robustness of the RTS classification under the output oriented DEA method. Note that an IRS DMU (under the output-oriented DEA method) must be termed as IRS by the input oriented DEA method. Therefore, one only needs to check the CRS and DRS supply chain processes in the current study. Using the input oriented approach, we discover only two DRS supply chain processes in replenishment cycle (DMUs 2, 4, 6 and 9) and seven DRS (DMUs 1, 3, 4, 5, 6, 7, and 9) in the manufacturing cycle. These results indicate that (i) in general; the RTS classification under different process cycle is independent of the orientation of DEA model; and (ii) there are serious input deficiencies in manufacturing cycle at the current usage quantities derived from engineering and process design. Given the fact that supply chains are assigned different efficiencies in case of CRS and VRS assumptions, i.e., using CCR models and BCC models, we can distinguish two different kinds of efficiencies Technical and Scale Efficiencies. The CCR model (without the convexity constraint) estimates the gross efficiency of a supply chain. This efficiency comprises technical efficiency and scale efficiency. Technical efficiency describes the efficiency in converting inputs to outputs, while scale efficiency recognizes that economy of scale cannot be attained at all scales of production, and that there is one most productive scale size, where the scale efficiency is maximum at 100%. The multi-stage VRS model takes into account the variation of efficiency with respect to the scale of operation, and hence measures pure Technical Efficiency. Note that while only DMU 2 is assigned 100% efficiency in the case of the CRS assumption, DMU 6 is considered 100% efficient in case of the VRS assumption. This indicates that the inefficiencies assigned to DMU 2 in case of the CRS assumption are purely due to their scales of operation.

Although a number of observations on supply chain cycles are efficient, only one supply chain performance (DMU 2) is efficient, i.e., the observation 2 represents the best practice of the supply chain system. Note that, all the supply chain cycles are efficient. Note that individual supply chain process efficiency is greater than the overall supply chain efficiency score, indicating that supply chain system could achieve more input savings.

Model 24 yields optimal values on the performance measures for supply chain to reach the best practice. Consider DMU 4 in **Table 5**. Since customer order cycle is efficient, no adjustments for measures related to the customer cycle are required. However, in order to reach the best practice, the replenishment, manufacturing, and procurement cycles should reduce their direct input. In addition, the procurement and manufacturing cycles should reach an agreement on the procurement price of raw materials to increase the revenue of procurement cycle. The fill rate of replenishment cycle should be increased. This solution indicates that based upon the best practice, the replenishment cycle should be able to maintain a fill rate of 90% while the manufacture reduces its shipment to the distributor of replenishment cycle.

Some supply chains may choose to operate with high cost and high availability while others are lean with lower levels of service. The notion of DEA efficiency provides an approach for efficiency measurement of supply chain and its processes. Multi-echelon VRS DEA models makes it clear that two supply chains may have different input-output mix yet both may be efficient. This model enables supply chain processes to collectively improve the supply chain performance. Through the use of the proposed models, any supply chains can find ways to achieve best-practice performance and to gain competitive edge.

4. Conclusion

Recent literature has examined a particular form of network structure, namely, where the DMU is a two-stage serial process in which the outputs from the first stage are intermediate variables that serve as inputs to the second stage. The current article extends this idea to include those situations where the overall process can be decomposed into product of the efficiencies of four stages. Therefore, we propose two models of efficiency decomposition that deals with the assumption of variable returns to scale (VRS). The proposed models, i.e., multi-stage DEA variable returns to scale (VRS) models that we have developed, adopt an alternative view of efficiency decomposition four-echelon supply chain structure. Our approach extends and generalizes the [8] and [10] two-stage models to four-echelon supply chain model with inclusion of the supply chain process concept in using inputs and outputs.

The analysis of the process cycles of 11 supply chains using the proposed DEA models shows that close to 45% of the supply chains were inefficient in four process cycles namely—customer order cycle, replenishment process cycle, manufacturing cycle and procurement cycle. Further, most supply chains exhibited DRS in manufacturing cycle and procurement cycle, while some of them exhibited IRS in customer order cycle and replenishment process cycle. This suggests that up-stream components of the supply chain may have a negative effect on finished product cycle time and end item.

We developed the multi-stage DEA models to evaluate the efficiency of supply chains. In these models, firms' production processes in multi-stages are interrelated. The empirical application shows that using conventional DEA models could lead to significantly biased evaluation results in multi-stage production situations. We also show that breaking down the production processes of supply networks for evaluation can generate more practical insights in how to improve the supply network performance, either in terms of technical or scale efficiencies.

The multi-echelon DEA models developed in this paper can be applied to a wide range of practical situations, including evaluating the effect of investments in IT systems and environmental improvements, human resources and the pollution effect etc. Future studies can deal with evaluating panel and longitudinal performance and efficiency changes of firms (e.g., [31–33]). The multi-stage DEA model can benefit these studies by providing a more accurate estimation of firms' performance over time. In the multi-stage DEA models the assumption of sequential flow of inputs and outputs may be relaxed to give rise to a complex model that can best fit the real world scenario.

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